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Bose–Einstein condensation and the Casimir effect for an ideal Bose gas confined between two slabs

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Abstract

We study the Casimir effect for a 3D system of ideal Bose gas in a slab geometry with a Dirichlet boundary condition. We calculate the temperature (T) dependence of the Casimir force below and above the Bose–Einstein condensation temperature (T_c). At $T \leq T_c$ the Casimir force vanishes as $[\frac{T}{T_c}]^{3/2}$. For $T \gtrsim T_c$ it weakly depends on temperature. For $T \gg T_c$ it vanishes exponentially. At finite temperatures this force for thermalized photons in between two plates has a classical expression which is independent of \hbar . At finite temperatures the Casimir force for our system depends on \hbar .

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Vacuum fluctuation of the electromagnetic field would cause an attractive force between two closely spaced parallel conducting plates. This phenomenon is called the Casimir effect and this force is called the Casimir force [1–3]. In the original paper [1] the Casimir force at zero temperature ($T = 0$) was defined as

$$F_c(L) = -\frac{\partial}{\partial L}[E(L) - E(\infty)] \quad (1)$$

where $E(L)$ is the ground-state energy (i.e. the vacuum energy) of the electromagnetic field in between the two conducting plates separated at a distance L . This force has been measured experimentally [4]. However, the Casimir effect can be generalized [5] for any range of temperature and for any dielectric substance between two dielectric plates. It has also been generalized for thermodynamical systems [6]. The Casimir force for this kind of system has recently been measured [7]. At finite temperature T , the definition of the Casimir force is generalized as [8–10]

$$F_c(T, L) = -\frac{\partial}{\partial L}[\Omega_T(L) - \Omega_T(\infty)] \quad (2)$$

where $\Omega_T(L)$ is the grand potential of the system confined between two plates separated at a distance L .

We consider the Casimir effect for a thermodynamical system of Bose gas between two infinite slabs. The geometry of the system on which some external boundary condition can be imposed is responsible for the Casimir effect. Thermalized photons (massless bosons) in between two conducting plates of area A at temperature T gives rise to the Casimir pressure [11–15]

$$\begin{aligned} \frac{F_c(L)}{A} &\sim -\frac{\pi^2 \hbar c}{240L^4} \left[1 + \frac{16(kT)^4 L^4}{3(\hbar c)^4} \right] && \text{for } \frac{\pi \hbar c}{kTL} \gg 1 \\ &\sim -\frac{kT \zeta(3)}{8\pi L^3} && \text{for } \frac{\pi \hbar c}{kTL} \rightarrow 0 \end{aligned} \quad (3)$$

where k is the Boltzmann constant, c is the velocity of light and L is the separation of the parallel plates. At $T \rightarrow 0$, the Casimir pressure becomes $-\frac{\pi^2 \hbar c}{240L^4}$ and it is only the vacuum fluctuation which contributes to the Casimir pressure. At high temperature, i.e. for $\frac{\pi \hbar c}{kTL} \rightarrow 0$, the Casimir force for photon gas goes as L^{-3} and has a purely classical expression independent of \hbar .

Let us consider a Bose gas confined between two infinitely large square shaped hard plates of area A . The plates are along the x - y plane and they are separated along the z -axis by a distance L . For the slab geometry, $\sqrt{A} \gg L$. We consider that our system is in thermodynamic equilibrium with its surroundings at temperature T . At this temperature the thermal de Broglie wavelength of a single particle of mass m is $\lambda = \sqrt{\frac{\pi \hbar^2}{2mkT}}$. In the thermodynamic limit, $\frac{\lambda}{L} \ll 1$. For this system the single particle energy is $\epsilon(p_x, p_y, j) = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{\pi^2 \hbar^2 j^2}{2mL^2}$, where p_x and p_y are the momentum along the x -axis and the y -axis, respectively and $j = 1, 2, 3, \dots$. However, in the thermodynamic limit the single particle energy can be written as $\epsilon(p_x, p_y, p_z) = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m}$, where p_z is the momentum along the z -axis.

Considering the thermodynamic limit the total number of thermally excited particles can be written as

$$N_T = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{e^{\frac{\frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} - \mu}{kT}} - 1} \frac{V dp_x dp_y dp_z}{[2\pi \hbar]^3} \quad (4)$$

where μ is the chemical potential and V is the volume of the system. Bose condensation temperature (T_c) is defined as a temperature where all the particles are thermally excited and below that temperature a macroscopic number of particles come to the ground state [16–18]. At $T \leq T_c$ the chemical potential goes to the ground-state energy. So

$$\begin{aligned} N &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{e^{\frac{\frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m}}{kT_c}} - 1} \frac{V dp_x dp_y dp_z}{[2\pi \hbar]^3} \\ &= \frac{V}{[2\pi \hbar]^3} (2\pi mkT_c)^{3/2} \sum_{i=1}^{\infty} \frac{1}{i^{3/2}} \\ &= \frac{V}{8} \frac{1}{\lambda_c^3} \zeta(3/2) \\ &= \frac{AkT_c m}{2\pi \hbar^2} \frac{L}{2\lambda_c} \zeta(3/2) \end{aligned} \quad (5)$$

where $\lambda_c = \sqrt{\frac{\pi\hbar^2}{2mkT_c}}$. Now from equation (5) we have

$$T_c = \frac{1}{k} \left[\frac{2\pi\hbar^2}{m} \right] \left[\frac{N}{V\zeta(3/2)} \right]^{\frac{2}{3}}. \quad (6)$$

Let us now introduce the finite size correction. The ground-state energy of our system is $[g = \frac{\pi^2\hbar^2}{2mL^2}]$. The average number of particles with energy $\epsilon_{p_x, p_y, j}$ is given by

$$\frac{1}{e^{\left\{ \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{\pi^2\hbar^2(j^2-1)}{2mL^2} - \mu' \right\} / kT} - 1}$$

where $\mu' = (\mu - g) \leq 0$ for bosons. At and below the condensate temperature $\mu' \rightarrow 0$. For this bosonic system we have the grand potential

$$\begin{aligned} \Omega &= \Omega(A, L, T, \mu') \\ &= kT \sum_{j=1}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\frac{A dp_x dp_y}{[2\pi\hbar]^2} \log \left[1 - e^{-\frac{\left(\frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{\pi^2\hbar^2(j^2-1)}{2mL^2} - \mu' \right)}{kT}} \right] \right]. \end{aligned} \quad (7)$$

Replacing j by $(j' + 1)$, we recast the above equation as

$$\begin{aligned} \Omega(A, L, T, \mu') &= -kT \sum_{i=1}^{\infty} \int_{p_x=0}^{\infty} \int_{p_y=0}^{\infty} \sum_{j'=0}^{\infty} \frac{A dp_x dp_y}{[2\pi\hbar]^2} \\ &\quad \times \frac{e^{\frac{i\mu'}{kT}} e^{-\frac{ip_x^2}{2mkT}} e^{-\frac{ip_y^2}{2mkT}} e^{-i(\pi(\frac{j'}{L})^2[j'^2+2j'])}}{i} \end{aligned} \quad (8)$$

where $\lambda = \sqrt{\frac{\pi\hbar^2}{2mkT}}$. Integrating over p_x and p_y we get

$$\begin{aligned} \Omega(A, L, T, \mu') &= -\frac{AkT}{(2\pi\hbar)^2} [2\pi mkT] \sum_{i=1}^{\infty} \sum_{j'=0}^{\infty} \frac{e^{i\mu'/kT}}{i^2} \left[e^{-\frac{\pi i \lambda^2 (j'^2+2j')}{L^2}} \right] \\ &= -\frac{A(kT)^2 m}{2\pi\hbar^2} \sum_{i=1}^{\infty} \sum_{j'=0}^{\infty} \frac{e^{i\mu'/kT}}{i^2} \left[e^{-\frac{\pi i \lambda^2 j'^2}{L^2}} \right] \\ &\quad \times \left[1 - 2j' \frac{\pi i \lambda^2}{L^2} + 2j'^2 \left(\frac{\pi i \lambda^2}{L^2} \right)^2 - \frac{4}{3} j'^3 \left(\frac{\pi i \lambda^2}{L^2} \right)^3 + \dots \right]. \end{aligned} \quad (9)$$

Since $\frac{\lambda}{L} \ll 1$, the higher order terms of the above series would not contribute significantly. From the Euler–Maclaurin summation formula we convert the summation over j' to integration. So from equation (9) we have

$$\begin{aligned} \Omega(A, L, T, \mu') &= -\frac{A(kT)^2 m}{2\pi\hbar^2} \sum_{i=1}^{\infty} \frac{e^{i\mu'/kT}}{i^2} \left[\left(\int_0^{\infty} e^{-\frac{\pi i \lambda^2 j'^2}{L^2}} dj' + \frac{1}{2} \right) \right. \\ &\quad - 2 \frac{\pi i \lambda^2}{L^2} \left(\int_0^{\infty} j' e^{-\frac{\pi i \lambda^2 j'^2}{L^2}} dj' - \frac{1}{12} \right) + 2 \left[\frac{\pi i \lambda^2}{L^2} \right]^2 \left(\int_0^{\infty} j'^2 e^{-\frac{\pi i \lambda^2 j'^2}{L^2}} dj' \right) \\ &\quad \left. - \frac{4}{3} \left[\frac{\pi i \lambda^2}{L^2} \right]^3 \left(\int_0^{\infty} j'^3 e^{-\frac{\pi i \lambda^2 j'^2}{L^2}} dj' + \frac{6}{720} \right) + \dots \right]. \end{aligned} \quad (10)$$

Collecting the leading terms from the above equation (10) we can write

$$\begin{aligned} \Omega(A, L, T, \mu') &= -\frac{A(kT)^2 m}{2\pi\hbar^2} \sum_{i=1}^{\infty} \frac{e^{i\mu'/kT}}{i^2} \left[\frac{L}{2\lambda i^{1/2}} - \frac{1}{2} + \frac{\pi}{2} i^{1/2} \frac{\lambda}{L} + O\left(\left[\frac{\lambda}{L}\right]^2\right) \right] \\ &= -\frac{A(kT)^2 m}{2\pi\hbar^2} \left[\frac{L}{2\lambda} g_{\frac{5}{2}}(z) - \frac{1}{2} g_2(z) + \frac{\pi\lambda}{2L} g_{\frac{3}{2}}(z) \right] \end{aligned} \tag{11}$$

where $z = e^{\mu'/kT}$ is the fugacity and $g_l(z) = z + \frac{z^2}{2^l} + \frac{z^3}{3^l} + \dots$ is the Bose–Einstein function. From the above equation we get the total number of particles as

$$\begin{aligned} N &= -\frac{\partial\Omega}{\partial\mu'} \\ &= \frac{AkTm}{2\pi\hbar^2} \left[\frac{L}{2\lambda} g_{\frac{3}{2}}(z) - \frac{1}{2} g_1(z) + \frac{\pi\lambda}{2L} g_{\frac{1}{2}}(z) \right]. \end{aligned} \tag{12}$$

In the thermodynamic limit of a system, as $T \leq T_c, z \rightarrow 1$. For a finite system this cannot happen, otherwise the correction terms in the above expression would be infinite. Instead at $T \gtrsim T_c, z \sim 1$. Taking only the first correction term in equation (12) we have $g_1(z) = -\ln(1-z) = \left[N'(T)g_{\frac{3}{2}}(z) - N\zeta(3/2) \right] \frac{L}{\lambda N'(T)} = -\ln \Delta z$, where $N'(T) = \frac{AkTm}{2\pi\hbar^2} \left[\frac{L}{2\lambda} \zeta(3/2) \right]$ and $\Delta z = 1 - z$ is a small change in the fugacity at $T \gtrsim T_c$. Now putting $z = 1$ in the expression of Δz , we get $\Delta z = e^{-\Delta N \zeta(3/2) L / (N'(T)\lambda)}$, where $\Delta N = N'(T) - N$. We see that in the thermodynamic limit ($L \rightarrow \infty$) $\Delta z = 0$ and when L is finite such that $L/\lambda \gg 1$, we have $z \sim 1$ at $T \gtrsim T_c$.

Let us now calculate the Casimir force. At $T \leq T_c$ we put $\mu' \rightarrow 0$ or $z \rightarrow 1$. So from equation (11) we have

$$\Omega(A, L, T, 0) = -\frac{A(kT)^2 m}{2\pi\hbar^2} \left[\frac{L}{2\lambda} \zeta(5/2) - \frac{1}{2} \zeta(2) + \frac{\pi}{2} \zeta(3/2) \frac{\lambda}{L} \right]. \tag{13}$$

Here the first term of equation (13) is

$$\Omega_b = -\frac{A(kT)^2 m}{2\pi\hbar^2} \left[\frac{L}{2\lambda} \zeta(5/2) \right]. \tag{14}$$

It is the bulk term of the grand potential. From our consideration of the thermodynamic limit $\frac{AL}{N} = \text{constant}$. So $\Omega_T(\infty) = \Omega_b$. The second term of equation (13) is $(\Omega_s) = \frac{A(kT)^2 m}{2\pi\hbar^2} \left[\frac{1}{2} \zeta(2) \right]$. It is the surface term of the grand potential. This surface term would have been different if we had evaluated the grand potential with a Neumann boundary condition [10]. The third term of equation (13) is the Casimir term of the grand potential. We call it the Casimir potential. Now putting $N = \frac{AkT_c m}{2\pi\hbar^2} \frac{L}{2\lambda_c} \zeta(3/2)$ in equation (13) we find the Casimir potential as

$$\begin{aligned} \Omega_c &= -\frac{A(kT)^2 m L}{2\pi\hbar^2} \frac{1}{\lambda} \zeta(3/2) \pi \left(\frac{\lambda}{L} \right)^2 \\ &= -NkT\pi \left(\frac{\lambda}{L} \right)^2 \left(\frac{T}{T_c} \right)^{3/2}. \end{aligned} \tag{15}$$

Putting $\lambda = \sqrt{\frac{\pi\hbar^2}{2mkT}}$ in equation (15) we have

$$\Omega_c = -N \left[\frac{T}{T_c} \right]^{3/2} \frac{\pi^2 \hbar^2}{2mL^2}. \tag{16}$$

From equations (2) and (14) we have

$$F_c(T, L) = -\frac{\partial}{\partial L} \Omega_c. \tag{17}$$

For $T \leq T_c$, from equations (16) and (17) we have the expression of Casimir force as

$$F_c(T, L) = -N \left[\frac{T}{T_c} \right]^{3/2} \frac{\pi^2 \hbar^2}{mL^3}. \quad (18)$$

This expression for the Casimir force shows that at finite temperatures the force depends on \hbar .

Above the condensation temperature $\mu' < 0$ or $z < 1$. However, for $T \gtrsim T_c$, $z \sim 1$. So at $T \gtrsim T_c$, from equations (11) and (5) with trivial manipulation we get the Casimir potential as

$$\begin{aligned} \Omega_c &= -\frac{A(kT)^2 m \pi \lambda}{2\pi \hbar^2} \frac{\pi \lambda}{2L} g_{\frac{3}{2}}(z) \\ &\approx -\pi N k T \left(\frac{\lambda}{L} \right)^2 \left(\frac{T}{T_c} \right)^{3/2}. \end{aligned} \quad (19)$$

In the above equation for $T \gtrsim T_c$, we put $z = 1$. However, as $\frac{T}{T_c}$ increases z decreases. So, for $T \gtrsim T_c$ the Casimir potential weakly depends on temperature. Putting $T = T_c + \Delta T$ in equation (19) and from the definition of the Casimir force we have

$$F_c(T, L) \approx -N \frac{\pi^2 \hbar^2}{mL^3} \quad (20)$$

where $0 < \frac{\Delta T}{T_c} \ll 1$. For $T \gtrsim T_c$, the Casimir force weakly depends on temperature.

Let us now calculate the Casimir force at $T \gg T_c$. At these temperatures $z \ll 1$. So we can approximately write $g_i(z) \approx z + \frac{z^2}{2i}$. From the first term of equation (12) we have $g_{\frac{3}{2}}(z) = \frac{N 2\pi \hbar^2 2\lambda}{A k T m L} \approx z + z^2/(2\sqrt{2})$. For this range of temperatures we can write $e^{\mu/kT} = z e^{-\pi \lambda^2/L^2} \approx z$. For convenience, we replace μ' by μ and recast equation (7) as [10]

$$\begin{aligned} \Omega &= \Omega(A, L, T, \mu) \\ &= -kT \sum_{j=1}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\frac{A dp_x dp_y}{[2\pi \hbar]^2} \log \left[1 - e^{-\frac{-(\frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{\pi^2 \hbar^2 j^2}{2mL^2} - \mu)}{kT}} \right] \right] \\ &= -\frac{A(kT)^2 m}{2\pi \hbar^2} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{e^{\mu i/kT}}{i^2} e^{-i\pi \lambda^2 j^2/L^2} \\ &= -\frac{A(kT)^2 m}{2\pi \hbar^2} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{e^{\mu i/kT}}{i^2} \left(\frac{L}{2\lambda i^{\frac{1}{2}}} - \frac{1}{2} + \frac{L}{\lambda i^{\frac{1}{2}}} e^{-\frac{\pi L^2 j^2}{i\lambda^2}} \right) \end{aligned} \quad (21)$$

where we use the formula

$$\sum_{n=-\infty}^{\infty} e^{-\pi a n^2} = \frac{1}{\sqrt{a}} \sum_{n=-\infty}^{\infty} e^{-\pi n^2/a}.$$

From the above equation we choose the Casimir potential as

$$\Omega_c = -\frac{A(kT)^2 m}{2\pi \hbar^2} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{e^{\mu i/kT}}{i^{5/2}} \frac{L}{\lambda} e^{-\frac{\pi L^2 j^2}{i\lambda^2}}.$$

For $\frac{T}{T_c} \rightarrow \infty$, in the expression of the above Casimir potential we can put $e^{\mu/kT} = z \ll 1$ and can take $i = 1$ and $j = 1$ as the leading term to contribute in the Casimir potential. So, for $T \gg T_c$ the Casimir potential is

$$\begin{aligned} \Omega_c &= -\frac{A(kT)^2 m}{2\pi \hbar^2} \frac{L}{\lambda} \frac{e^{\mu/kT}}{1^{5/2}} e^{-\pi L^2/\lambda^2} \\ &= -2N k T e^{-\frac{2mL^2 k T}{\hbar^2}} \end{aligned} \quad (22)$$

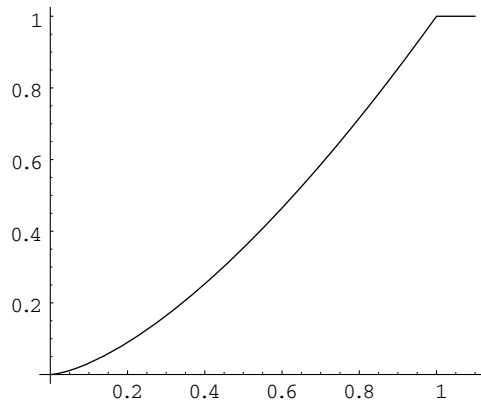


Figure 1. Casimir force ($F_c(T, L)$) versus temperature (T) plot. $F_c(T, L)$ is in units of $N \frac{\pi^2 \hbar^2}{mL^3}$ and T is in units of T_c . Equation (18) corresponds to the range $\frac{T}{T_c} \leq 1$. Equation (20) corresponds to the range $1 < \frac{T}{T_c} \leq 1.1$.

where we put $e^{\mu/kT} = z \approx g_{\frac{3}{2}}(z) = \frac{N2\pi\hbar^22\lambda}{AkTmL}$. From equation (22), for $T \gg T_c$ we have the Casimir force as

$$\begin{aligned}
 F_c(T, L) &= -\frac{\partial \Omega_c}{\partial L} \\
 &= -\frac{8N(kT)^2 mL}{\hbar^2} e^{-\frac{2mL^2kT}{\hbar^2}}.
 \end{aligned}
 \tag{23}$$

Now we see that in the classical limit ($T \gg T_c$) the Casimir force vanishes as e^{-kT} . This exponential behavior was also obtained in [10]. This expression of Casimir force is maximized at an optimized temperature $T_{\max} = \frac{\hbar^2}{kmL^2}$. At this optimized temperature $\frac{\lambda}{L}$ is of the order of unity. Since equation (23) is obtained for $T \gg T_c$ and in this range of temperature $\frac{\lambda}{L}$ is much less than 1, this optimization has no physical significance. But, it is interesting to see that the Casimir force for $T \gg T_c$ is long ranged (exponential decay) and for $T \leq T_c$ is short ranged (power law decay) [10].

The changes of the Casimir force with temperature for the range $0 < T \leq T_c$ and for the range $T \gtrsim T_c$ are shown in figure 1.

That vacuum fluctuation causes Casimir force is well known [1, 4, 19]. Critical fluctuations also cause Casimir force [7, 9]. The Casimir force calculated here is neither due to vacuum fluctuations nor due to critical fluctuations. It is due to quantum fluctuations. These fluctuations are associated with the commutator algebra of the position and momentum operator as well as with the commutator algebra of the bosonic annihilation operator (\hat{a}_i) and creation operator (\hat{a}_i^\dagger) such that $[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{i,j}$, where i, j represent the single particle energy states. At $T \ll T_c$ almost all the particles come down to the ground state. The quantum fluctuations die out due to the macroscopic occupation of particles in a single state. That is why the Casimir force dies out at $T \ll T_c$. At $T \gg T_c$ the Bose–Einstein statistics becomes classical Maxwell–Boltzmann statistics and thermal fluctuations dominates over the quantum fluctuations. For this reason the Casimir force dies out at $T \gg T_c$. Although the problem of the Casimir effect on this system was attacked by many authors [10, 20, 21], yet our advantage is the simplicity of calculation where the machinery of critical fluctuations is not needed. Since the Casimir force is measured with a plate separation of a few micrometers, it is difficult to put Bose condensate within this plate separation. However, we are dealing with a purely

theoretical issue of the effect of Bose–Einstein condensation on the Casimir effect. For our system, the reduction of the thermodynamic Casimir force with the $T^{3/2}$ law will show the signature of the Bose–Einstein condensation. To put the Bose condensate between the two walls, we must have to take perfectly reflecting walls. Otherwise the Bose particles will collide with the walls and lose their energy to form molecules, and eventually it will become a solid. However, there is a problem with the existence of perfectly reflecting walls because the cold atoms interact with the walls by the van der Waals force.

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